Lecture: Binomial Ideals, Combinatorics, and Statistics

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Binomial ideals are ideals generated by polynomials with at most two terms. The special form of their generators makes it possible to translate problems like Gröbner basis computations and primary decomposition into an adapted combinatorial language, leading to algorithms which are much faster than the generic algorithms of computational commutative algebra [4]. On the other hand the class of binomial ideals is still large enough to contain many interesting and relevant examples. In this lecture we will mainly focus on applications in "algebraic statistics" [2].

On one hand there is the theory of *toric varieties*, which correspond to binomial prime ideals. Most constructions of algebraic geometry (like divisors) have a correspondence in the polyhedral fan associated to the variety. This allows a visualization in concrete geometric terms. In statistics toric varieties appear in the form of exponential families, which are important statistical models. Generating sets of the toric ideals are used in model selection tasks and in integer programming.

On the other hand there is binomial primary decomposition, which enters statistics when modeling conditional independences among random variables. Independence statements give equations, and primary decompositions of the respective ideals give insights into structural zeros of these models.

For this lecture basic knowledge of commutative algebra (first few chapters of [3]) and algebraic geometry [1] is certainly helpful, but we will repeat the necessary theory where needed. There will be exercises accompanying the lecture, however, there will be no marks, no exams, and no "Scheine".

Dates: The lecture will be given once weekly in room A2 of the Max-Planck-Institute for Mathematics, Inselstr. 22, and start in the 3rd week of May. The exact coordinates will be negotiated by email shortly before the lecture starts. **Suggestion: Tuesday, 5:15 pm from May 11th**. Please e-mail to **kahle@mis.mpg.de** to be updated.

References

[1] David A. Cox, John B. Little, and Donal O'Shea, *Ideals, varieties, and algorithms*, second ed., UTM, Springer, New York, 1996.

[2] Mathias Drton, Bernd Sturmfels, and Seth Sullivant, *Lectures on algebraic statistics*, Oberwolfach Seminars, vol. 39, Springer, Berlin, 2009, A Birkhäuser book.

[3] David Eisenbud, *Commutative algebra: with a view toward algebraic geometry*, GTM, vol. 150, Springer Verlag, New York, 1995.

[4] David Eisenbud and Bernd Sturmfels, *Binomial ideals*, Duke Mathematical Journal **84** (1996), no. 1, 1–45.